

Nationality		No.	
Name	(Please print full name, underlining family name)		

Marks	
-------	--

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) Let  $a$  and  $b$  be an integer part and an decimal fraction of  $\sqrt{7}$ , respectively. Then the integer part of  $\frac{a}{b}$  is  $\boxed{[1-1]}$ .

(2) Consider a cone with a diameter of 12 and a height of 8. The volume of an inscribed sphere in the cone is  $\boxed{[1-2]}$ .

(3)  $5^{29}$  is an integer with  $\boxed{[1-3]}$  places by assuming that  $\log_{10} 2 = 0.3010$ .

(4) There is a circle with a radius of 2 where the center is at the origin and a line  $3x + 4y - 12 = 0$  in the plane. The minimum distance between a point on the circle and a point on the line is  $\boxed{[1-4]}$ .

(5) If the series  $\{a_k\}$  satisfies that  $a_1 = 1, a_2 = 2$ , and  $a_k - 4a_{k-1} + 3a_{k-2} = 0$  ( $k \geq 3$ ), then  $a_k = \frac{1 + \boxed{[1-5]}}{\boxed{[1-6]}}$  ( $k \geq 1$ ).

(6) Let  $f(x) = ax + b$  be a linear function. If the equation

$$\int_{-m/2}^m f(x)dx = \frac{m(m+1)}{2}$$

holds for any positive  $m$ , then  $f(x) = \frac{\boxed{[1-7]}x + \boxed{[1-8]}}{3}$ .

**2.** Consider a semicircle with a diameter  $AB$  where the length is 4, and a point  $C$  on the circular arc. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) The maximum of the area of the triangle  $ABC$  is  $\boxed{[2-1]}$  .

(2) If the area of the triangle  $ABC$  is a half of the maximum and point  $C$  is nearer to point  $A$  than point  $B$ , then the angle  $\angle CAB$  is  $\boxed{[2-2]}$  .

**3.** Consider a function

$$y = \left(x^3 + \frac{1}{x^3}\right) - 6\left(x^2 + \frac{1}{x^2}\right) + 3\left(x + \frac{1}{x}\right)$$

defined in  $x > 0$ .

(1) Letting  $t = x + \frac{1}{x}$  gives

$$y = \boxed{[3-1]}t^3 + \boxed{[3-2]}t^2 + \boxed{[3-3]}t + \boxed{[3-4]}.$$

Here it holds that

$$t = x + \frac{1}{x} \geq \boxed{[3-5]}.$$

(2) When  $t = \boxed{[3-6]}$ , that is,  $x = \boxed{[3-7]}$ ,  $y$  has the minimum value  $\boxed{[3-8]}$ .