MATHEMATICS (B) (2016)

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

(1) If $\log_3 6 - \log_9 x = \dfrac{1}{2}$, then $x =$ [ ].

(2) If α, β are numbers satisfying $0 < \alpha < \dfrac{\pi}{4}$, $0 < \beta < \dfrac{\pi}{4}$, $\alpha + \beta = \dfrac{\pi}{4}$, it follows that $(\tan\alpha + 1)(\tan\beta + 1) =$ [ ].

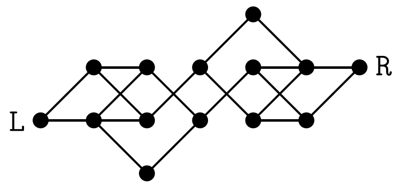
(3) When $x + y = \dfrac{2\pi}{3}$, $x \geq 0$, $y \geq 0$, the maximum of $\sin x + \sin y$ is [①], and the minimum of that is [②].

(4) On the basis of the premises and the conclusions ①, ②, ③ below, fill in the lefthand blanks with 1 if the corresponding conclusion is logically derived from the premises, and with 0 if it is not.

> **Premises**: There are several three-digit numbers NML each digit of which is either 1 or 2. There are some numbers with $N = 1$ and other numbers with $N = 2$. If $M = 2$, then $N = 2$. And, if $L = 1$, then $N = 2$.

①	**Conclusion** ①: If $N = 1$, then $M = 1$.
②	**Conclusion** ②: There are no numbers with $M = 1$ and $N = 2$.
③	**Conclusion** ③: There are no numbers with $M = 1$ and $L = 1$.

(5) Consider the following diagram that consists of vertices (●) and edges (╱ or — or ╲); crosses (╳) are pairs of edges whose crossing points are not vertices.



Suppose that one can move from one vertex to another if, and only if, the two vertices are connected by a unique common edge. The number of routes that one can take from the leftmost vertex `L` through 6 edges and 5 intermediate vertices to the rightmost vertex `R` is [ ].

2. Let r be a positive constant. Consider the cylinder $x^2 + y^2 \leq r^2$, and let C be the part of the cylinder that satisfies $0 \leq z \leq y$. Fill in the blanks with the answers to the following questions.

(1) Consider the cross section of C by the plane $x = t$ $(-r \leq t \leq r)$, and express its area in terms of r, t.

(2) Calculate the volume of C, and express it in terms of r.

(3) Let a be the length of the arc along the base circle of C from the point $(r, 0, 0)$ to the point $(r\cos\theta, r\sin\theta, 0)$ $(0 \leq \theta \leq \pi)$. Let b be the length of the line segment from the point $(r\cos\theta, r\sin\theta, 0)$ to the point $(r\cos\theta, r\sin\theta, r\sin\theta)$. Express a and b in terms of r, θ.

(4) Calculate the area of the side of C with $x^2 + y^2 = r^2$, and express it in terms of r.

(1) [ ]

(2) [ ]

(3) $a =$ ① [ ] $b =$ ② [ ]

(4) [ ]

3. Let a be a number with $a \neq 0$, $-1 < a < 1$, and b an arbitrary real number. Let $f(x) = ax + b$; moreover, let $f^1(x) = f(x)$, and $f^n(x) = f(f^{n-1}(x))$ $(n = 2, 3, 4, \dots)$. Fill in the blanks with the answers to the following questions.

(1) Express $f^n(x)$ $(n = 1, 2, 3, \dots)$ in terms of a, b, x, n.

(2) Express $\dfrac{f^n(x) - f^{n-1}(x)}{a^n}$ $(n = 2, 3, 4, \dots)$ in terms of a, b, x, n.

(3) Consider the curve $y = \dfrac{f^n(x) - f^{n-1}(x)}{a^n}$ $(n = 2, 3, 4, \dots)$ and the line $y = ax + b$. Find the intersection point $Q(x_n, y_n)$ of the curve and the line above, and express x_n, y_n in terms of a, b, n.

(4) Calculate the limit $\lim_{n \to \infty} f^n(x)$, and express it in terms of a, b, x.

(1) [ ]

(2) [ ]

(3) $x_n =$ ① [ ] $y_n =$ ② [ ]

(4) [ ]